Vector Cross Product: \( \vec{C} = \vec{A} \times \vec{B} \)

- The magnitude of \( \vec{C} \) is:
  \[ C = A \cdot B \sin \theta \]
  where \( \theta \) is the smaller of the two angles between \( \vec{A} \) and \( \vec{B} \).

- \( \vec{C} \) is perpendicular to the blue plane containing both \( \vec{A} \) and \( \vec{B} \). The direction is determined by the right hand rule (RHR).

**RHR**: The fingers of the right hand are curled in the direction from \( \vec{A} \) toward \( \vec{B} \) (through the smaller angle). The thumb points in the direction of \( \vec{C} \).
Vector Cross Product: \( \mathbf{D} = \mathbf{B} \times \mathbf{A} \)

RHR: The fingers of the right hand are curled in the direction from \( \mathbf{B} \) toward \( \mathbf{A} \) (through the smaller angle). The thumb points in the direction of \( \mathbf{D} \).

Note that:
\[
\mathbf{D} = -\mathbf{C}  \\
\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}
\]

Vector Cross Products of the Unit Vectors

\[
i \times j = k  \\
j \times k = i  \\
k \times i = j
\]

\[
j \times i = -k  \\
k \times j = -i  \\
i \times k = -j
\]

Note: \( i \times i = j \times j = k \times k = 0 \)
Vector Cross Product Example

\[ \vec{A} = 2\hat{i} - 3\hat{j} \quad \text{and} \quad \vec{B} = \hat{i} + 4\hat{j} \]

\[ \vec{A} \times \vec{B} = (2\hat{i} - 3\hat{j}) \times (\hat{i} + 4\hat{j}) = 2(\hat{i} \times \hat{i}) + 8(\hat{i} \times \hat{j}) - 3(\hat{j} \times \hat{i}) - 12(\hat{j} \times \hat{j}) \]

\[ \vec{A} \times \vec{B} = 11\hat{k} \]

The same result can also be obtained by finding the angle \( \theta \) between the vectors (132.3\(^\circ\)) and the magnitudes of \( A \) (3.606) and \( B \) (4.123). The magnitude of the vector product is \( AB\sin \theta = 11.0 \) and the direction is found from the RHR.

The Magnetic Field \( \vec{B} \)

Magnetic fields are created by moving charges. The calculation of these magnetic fields is discussed in the next chapter. In this chapter we assume that a magnetic field is known and we want to determine the force exerted, by that field, on moving charges.

The electric field was used in previous chapters to find the electric force exerted on a charge: \( \vec{F} = q\vec{E} \)

In a similar fashion we use the magnetic field, \( \vec{B} \), to find the magnetic force exerted on a moving charge: \( \vec{F} = q\vec{v} \times \vec{B} \)

Units for magnetic field: \( T \) (Tesla)
Earth’s Magnetic field: \( 0.5 \times 10^{-4} \) (T)
Strong magnet: \( 3 \) (T)
Magnetic Force on a Moving Charge

Magnetic Force on a moving positive charge:
- The velocity and magnetic field vectors are in the yellow plane.
- The magnetic force is: \( \mathbf{F} = (+q)v \times \mathbf{B} \).
- The magnitude of the force is: \( qvB\sin\theta \).
- The direction is obtained from the RHR.

Magnetic Force on a moving negative charge:
- The direction of \( \mathbf{v} \times \mathbf{B} \) is shown but this is not the direction of \( \mathbf{F} \).
- The magnetic force is: \( \mathbf{F} = (-q)v \times \mathbf{B} \) and is in the direction opposite to the vector \( \mathbf{v} \times \mathbf{B} \).

Magnetic Force on a Wire having a Current I

- Magnetic force on single charge = \( qv_d \times \mathbf{B} \)
- Total force on all charges = \( nAL(qv_d \times \mathbf{B}) \)
- In terms of the unit vector \( \hat{i} \) the drift velocity is \( v_d \). It is useful to define the length vector \( \mathbf{L} \) by: \( \mathbf{L} = \mathbf{L}\hat{i} \).

The total force becomes:
\[
\vec{F} = nALqv_d\hat{i} \times \mathbf{B} = (nqv_d)A(\mathbf{L}\hat{i} \times \mathbf{B}) = \mathbf{JL} \times \mathbf{B} = \mathbf{IL} \times \mathbf{B}
\]